Trajectory and Trajectory Experiments

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Many people think that bullets, shot or arrows travel in straight lines just like light. It does not take much shooting experience to realize that projectiles and light behave quite differently. The physics of projectiles (ballistics) is discussed at the end of this fact sheet for those interested. The main purpose of this material is to help you understand how trajectory enters into shooting.

Under normal conditions, light travels in straight lines. Changes in the density of substances it passes through may deflect it, but within substances of uniform density (like air) light travels in straight lines. Electromagnetic forces, like magnets can bend light, but it has no mass (weight).

Projectiles like bullets or arrows have mass. Thus, they respond to gravity under normal conditions. As soon as an arrow leaves the string or a bullet leaves the muzzle, it begins to fall, accelerating toward the earth under the influence of gravity. In fact, if an arrow or bullet was fired parallel to the surface of the earth on level ground, it would hit the earth at the same time as one dropped from the same height at the same time much farther away. In addition to their mass, projectiles are solid objects. Pushing them through a dense medium, like air, causes friction and turbulence. Both forces affect the projectile immediately. The projectile begins to slow down as soon as it leaves the string or the muzzle.

The slowing influence is cumulative until the projectile finally comes to rest.

These two factors combined cause projectiles to follow a curved flight path. If two straight lines are used to show the line of the bore or the resting position on the string and the line of sight, the line of flight (path of the projectile) would relate to those lines as shown below (Figure 1).

The curved flight path requires that the bore must be pointed above the line of sight to hit a distant object on the line of sight. If the sights are above the bore or the arrow, it must cross the line of sight twice, once while rising and a second time while falling toward the earth. The exact shape of the trajectory curve can be determined by complicated mathematics or by testing. Actual testing yields better results for a given shooter and his or her equipment and is more easily understood.
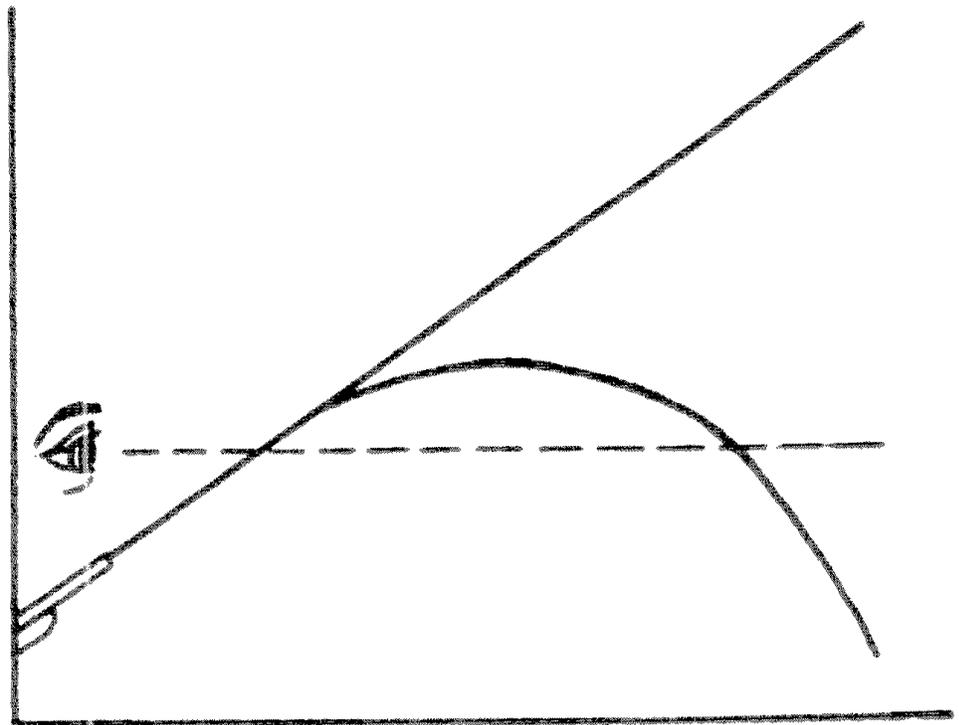

Figure 1. Relationships among line of sight, line of bore, flight path and drop.

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The projectile accelerates toward the earth at a rate of about 9.8 meters/second² (32 feet per second²). If the projectile was fired parallel to the ground, it would drop 4.9 centimeters (0.16 feet) in the first 0.1 second, 19.6 centimeters (0.64 feet) in 0.2 second, 122.5 centimeters (4 feet) in 0.5 second, and 490 centimeters (16 feet) in 1 second (Table 1). In 3 seconds, it would have dropped 44.1 meters (144 feet). If a projectile could travel at a constant velocity of 60.96 meters/second (200 feet/second), the trajectory would look like the graph in Figure 2. The actual flight path would curve more sharply downward, since the projectile would be slowing its horizontal motion as gravity pulls it to earth.

You can calculate the trajectory of your personal equipment as you have it set up using the worksheet attached. Shoot the same arrows throughout the experiment if possible. If not, shoot matched arrows with matched fletching. Leave the sight setting the same for all shots, and use the same aiming point for each shot and all distances. Measure from the aiming point to the center of the group of arrows and carefully record the distance above (+) or below (-) the point of aim for each distance. Stop shooting if the arrows are falling short (or nearly falling short) of the target. Plot the flight path of the arrows relative to the line of sight. The elevation angle can be calculated if desired by following these steps.

1. Measure the true draw length of the arrow at full draw (the distance from the anchor point to the arrow rest).
2. Measure the height of the sight pin above the arrow rest.
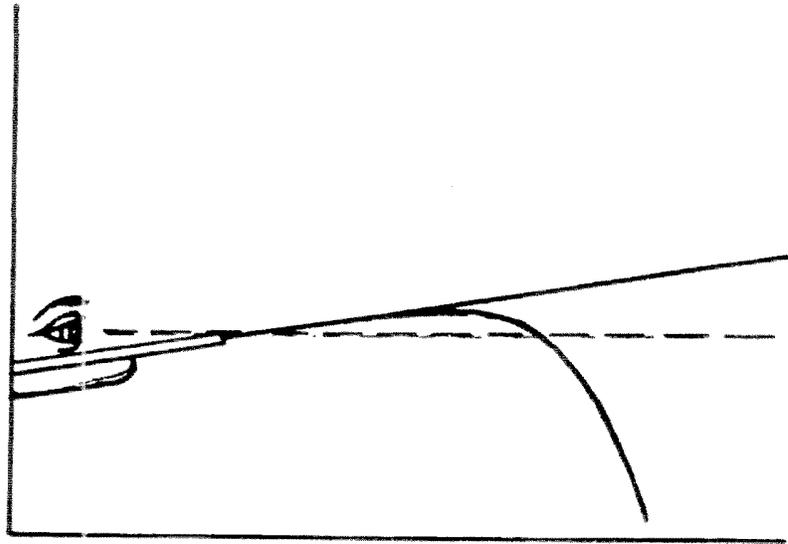3. Measure the height of the center of the eye above the bottom of the arrow shaft at the anchor point.

Figure 2. Trajectory of an idealized projectile traveling at a constant 60.96 meters/second (200 feet/second).

4. Subtract the sight pin height from the eye height.
5. Divide that length by the true draw length.
6. The dividend is the natural sine of the angle of elevation from the horizontal.
7. Use a sine table to locate the angle with that sine value.
8. Plot a horizontal line through the eye and sight to the center of the target.
9. Plot a line starting with the distance from the eye to the anchor point below the line of sight at the calculated angle of elevation above horizontal. This is the line of the shaft or the line of the bore.
10. Plot the true line of flight at the distances selected.

This same procedure can be used with rifle bullets, but a longer range is needed and the differences in impact points should be recorded in centimeters or inches rather than meters or feet.

For Those Who Want to Know More

Ballistics is the science of projectiles. It focuses on the dynamics of projectile flight and the energy stored and released by the projectile. One of the components of ballistics is the study of trajectory, the flight path of projectiles. In shooting, trajectory related the straight line of sight to a target with the curved flight line of the projectile. The ballistics of bullet or arrow trajectory involves complex concepts in physics and engineering.

Newton's first law, the law of inertia, states that objects tend to remain at rest or to travel in a straight line at a constant speed unless they are acted upon by an outside force. For projectiles, the forces include the energy that accelerates them initially, the acceleration of gravity, friction of the air and drag. Projectile mass, shape and even construction can influence those forces. Complex formulas have been developed to calculate the projectile's ability to retain its energy and velocity (speed

in a direction). Sectional density and ballistic coefficient are two measures of the "slipperiness" of a projectile.

A projectile starting at rest is accelerated by the stored energy of the limbs, air charge or expanding gases of the fired round. The internal ballistics (those inside the firearm bore or while the arrow is on the rest) limit the motion to horizontal and vertical vectors (components of the total velocity of the projectile). Under most conditions the horizontal velocity is greatest.

External ballistics are more complex, and they begin as soon as the projectile leaves the bore or the bow. The horizontal vector of velocity is measured by its horizontal speed toward the target. The vertical vector is measured by its speed upward, perpendicular to the surface of the earth. Both the horizontal and the vertical vectors of velocity begin to decrease immediately. Friction and turbulence in the air reduce the horizontal velocity. The pull of gravity reduces the vertical velocity. Note that a negative acceleration or velocity in the vertical component means the projectile is moving toward the earth rather than away from it. Wind currents or the rotation of the projectile may cause a lateral movement.

High initial and retained velocities result in a "flatter" trajectory. That is, the arc of the projectile from the shooter to the target is less peaked. The less time the projectile is in flight; the less outside forces influence it. The obvious conclusion is that "faster is better." However, obtaining optimum ballistic performance involves balancing competing factors rather than simple maximization of any one factor. Limits are imposed by the chemical structure of the

powder, strength of the materials used, mass of the firearm and the strength or recoil resistance of the shooter. Other factors, like barrel life, consistency in energy release, economics, projectile construction and many more reduce the upper limit toward some optimum value.

Changing a single factor of the internal ballistics can result in major changes in external ballistics. Altering the mass of a projectile results in changing its shape. Both sectional density (a measure of the mass divided by the diameter or basal area) and ballistic coefficient (a measure of the bullet's ability to overcome resistance of the air, which involves sectional density in its calculation) are related to the shape of the bullet. Optimizing mass and initial velocity with performance and terminal velocity involves many factors.

The results of changing bullet mass may be surprising. An empirical test could be done using bullets of similar shape and diameter but different weights. Thirty caliber bullets are available in flat-based spitzer shapes in weights from about 110 grains to 200 grains. Interested shooters could fire a test series with bullet weight and compare trajectory curves to determine the optimum bullet weight for that shape in their rifle.

Both momentum and energy are related to the velocity and mass of the projectile. Momentum is the product of the mass and the velocity. Kinetic energy (the energy of moving objects) is the product of the mass and the square of the velocity divided by two.

Projectile use is a major determining factor in balancing momentum and energy. When a projectile comes to rest, the remaining energy and momentum is translated into penetration and

shock. On target ranges bullets and arrows need only enough momentum and striking force to penetrate the target. The shot must have enough remaining energy and momentum to break clay targets. In hunting situations, small game arrows, shot and bullets kill by hydrostatic shock. The energy of the striking bullet displaces water in the tissue, causing immediate disruption of vital functions. Momentum is not critical, but kinetic energy is. Big game arrows kill by penetration and hemorrhage. Very little hydrostatic shock is produced, so momentum is much more important than kinetic energy. Big game bullets must combine shock with adequate penetration to reach vital areas. Considering the use of the projectile adds complexity to making ballistic decisions. This may explain the large array of arms and ammunition available today.

Many ballistic experiments can be tried by shooting sports participants. Most would be worthy of entry in science fair competitions under the categories of physics and engineering. Altering one factor at a time, such as sectional density or velocity, may offer easier explanations of the events taking place. For the shooter, more interested in performance on targets or game, the arms and ammunition companies have tables that can assist in selecting the appropriate combinations of factors to do the job at hand. Wise shooters will test the listed values for themselves using their own firearms, particularly where the shape of a trajectory curve is concerned. Their observed data is much more valuable than the theoretical data from the tables.

Table 1. Theoretical values for drop from the acceleration of gravity and distance traveled for idealized projectiles fired parallel to the surface of the earth at a constant velocity of 60.96 meters/second (200 feet/second) and 914 meters/second (3000 feet/second) is given below. The lower velocity corresponds to a very fast arrow. The higher velocity approximates a high velocity center-fire rifle cartridge.

Time	Distance Dropped		Distance Traveled @200ft.sec		Distance Traveled @3000ft.sec	
(sec)	(cm)	(ft)	(m)	(ft)	(m)	(ft)
0.1	4.7	0.16	6.1	20	91.4	300
0.5	19.6	0.64	12.2	40	182.8	600
0.3	44.1	1.44	18.3	60	271.2	900
0.4	78.4	2.56	24.4	80	365.6	1200
0.5	122.5	4.00	30.5	100	457.0	1500
0.6	176.4	5.76	36.6	120	584.4	1800
0.7	240.1	7.84	42.7	140	693.0	2100
0.8	313.6	10.24	48.8	160	731.0	2400
0.9	369.9	12.96	54.9	180	822.6	2700
1.0	490	16.00	61.0	200	914.0	3000
1.25	765.6	24.00	78.2	250	1142.5	3750
1.5	1102.5	36.00	91.4	300	1371.0	4500
1.75	1500.6	49.00	106.7	350	1599.5	5250
2.0	1960	64.00	121.9	400	1828.0	600

Trajectory Worksheet

This worksheet will help you determine trajectory of an arrow using your equipment. Distances and trajectory measures should be made in the same measurement units, either metric or English units. The suggested increment of distance for English units is about 5 feet. For metric units try an increment of 1 or 2 meters. Measure the distances above or below the line of sight in either inches or centimeters. If those distances become too great, feel free to convert them to feet or meters. Record the following information before you start.

Height of sight above arrow rest (bore) _____

Height of pupil above nock of drawn arrow _____

Distance* (m or ft)	Point of Impact**(cm or in)	+=above line of sight -=below line of sight